Límites Trigonométricos

Problemas

Ejercicio 1

- $$\lim_{x \to 0} \frac{\text{Sen } 5x}{\text{Sen } 3x} = \frac{\dfrac{\text{Sen } 5x}{x}}{\dfrac{\text{Sen } 3x}{x}} = \lim_{x \to 0} \frac{\dfrac{5}{5}\;\dfrac{\text{Sen } 5x}{X}}{\dfrac{3}{3}\;\dfrac{\text{Sen } 3x}{X}} =$$

$$\frac{\dfrac{5 \text{ Sen } 5x}{5X}}{\dfrac{3 \text{ Sen } 3x}{3x}} = \frac{5 \lim\limits_{x \to 0} \dfrac{\text{Sen } 5x}{5X}}{3 \lim\limits_{x \to 0} \dfrac{\text{Sen } 3x}{3x}} = \frac{5}{3} \Rightarrow \text{RPTA}$$

Ejercicio 2

$$\lim_{x\to 0} \frac{\operatorname{Sen} \Pi x}{\operatorname{Sen} 3 \Pi x} = \frac{\dfrac{\operatorname{Sen} \Pi x}{x}}{\dfrac{\operatorname{Sen} 3 \Pi x}{x}} = \frac{\dfrac{\Pi}{\Pi}\left(\dfrac{Sen\prod x}{x}\right)}{\dfrac{3\Pi}{\Pi}\left(\dfrac{Sen3\prod x}{x}\right)}$$

$$= \frac{\Pi \lim_{x\to 0} \dfrac{\operatorname{Sen} \Pi x}{\Pi x}}{3\Pi \lim_{x\to 0} \dfrac{\operatorname{Sen} \Pi x}{3\Pi x}} = \frac{\Pi}{3\Pi} = \frac{1}{3} \Rightarrow \text{RPTA}$$

Ejercicio 3

3. $$\lim_{x \to \infty} x \operatorname{sen} \frac{1}{x}$$

<u>Solución</u>:

$$\lim_{x \to \infty} x \operatorname{sen} \frac{1}{x} \qquad (\clubsuit)$$

Sea

$$t = \frac{1}{x}, \quad \Rightarrow \quad x \to \infty \Leftrightarrow t \to 0 \qquad (1)$$

$$t = \frac{1}{x} \Leftrightarrow x = \frac{1}{t} \qquad (2)$$

Sustituyendo (1) y (2) en ($\clubsuit$), se obtiene:

$$\lim_{x \to \infty} x \operatorname{sen} \frac{1}{x} = \lim_{t \to 0} \frac{1}{t} \operatorname{sen} t = \lim_{t \to 0} \frac{\operatorname{sen} t}{t} = 1 \qquad \{\mathbf{TL10}\}.$$

Ejercicio 4

4. $\lim\limits_{x \to \pi} \dfrac{\operatorname{sen} x}{x - \pi}$

Solución:

$$\lim_{x \to \pi} \frac{\operatorname{sen} x}{x - \pi} \quad (♣)$$

Sea

$$t = x - \pi, \;\Rightarrow\; x \to \pi \Leftrightarrow t \to 0 \quad (1)$$

$$t = x - \pi \Leftrightarrow x = t + \pi \quad (2)$$

Sustituyendo (1) y (2) en (♣), se obtiene:

$$\lim_{x \to \pi} \frac{\operatorname{sen} x}{x - \pi} = \lim_{t \to 0} \frac{\operatorname{sen}(t + \pi)}{t} = \lim_{t \to 0} \frac{-\operatorname{sen} t}{t} = -\lim_{t \to 0} \frac{\operatorname{sen} t}{t} = -1 \qquad \{\mathrm{TL10}\}.$$

Ejercicio 5

5. $\lim_{x \to \pi/2} \dfrac{1 - \operatorname{sen} x}{\frac{1}{2}\pi - x}$

<u>Solución:</u>

$$\lim_{x \to \pi/2} \frac{1 - \operatorname{sen} x}{\frac{1}{2}\pi - x} \qquad (\clubsuit)$$

Sea

$$t = \tfrac{1}{2}\pi - x, \ \Rightarrow \ x \to \tfrac{1}{2}\pi \Leftrightarrow t \to 0 \qquad (1)$$

$$t = \tfrac{1}{2}\pi - x \Leftrightarrow x = \tfrac{1}{2}\pi - t \qquad (2)$$

Sustituyendo (1) y (2) en ($\clubsuit$), se obtiene:

$$\lim_{x \to \pi/2} \frac{1 - \operatorname{sen} x}{\frac{1}{2}\pi - x} = \lim_{t \to 0} \frac{1 - \operatorname{sen}(\frac{1}{2}\pi - t)}{t} = \lim_{t \to 0} \frac{1 - \operatorname{sen}\left(-(t - \frac{1}{2}\pi)\right)}{t} = \lim_{t \to 0} \frac{1 - \left(-\operatorname{sen}(t - \frac{1}{2}\pi)\right)}{t},$$

$$\Rightarrow \quad \lim_{x \to \pi/2} \frac{1 - \operatorname{sen} x}{\frac{1}{2}\pi - x} = \lim_{t \to 0} \frac{1 + \operatorname{sen}(t - \frac{1}{2}\pi)}{t} = \lim_{t \to 0} \frac{1 - \cos t}{t} = 0 \qquad \{\text{TL11}\}.$$

Ejercicio 6

6. $\lim\limits_{x \to \pi/2} \dfrac{\frac{1}{2}\pi - x}{\cos x}$

Solución:

$$\lim_{x \to \pi/2} \frac{\frac{1}{2}\pi - x}{\cos x} \qquad (♣)$$

Sea

$$t = \tfrac{1}{2}\pi - x, \quad \Rightarrow \quad x \to \tfrac{1}{2}\pi \Leftrightarrow t \to 0 \qquad (1)$$

$$t = \tfrac{1}{2}\pi - x \Leftrightarrow x = \tfrac{1}{2}\pi - t \qquad (2)$$

Sustituyendo (1) y (2) en (♣), se obtiene:

$$\lim_{x \to \pi/2} \frac{\frac{1}{2}\pi - x}{\cos x} = \lim_{t \to 0} \frac{t}{\cos(\frac{1}{2}\pi - t)} = \lim_{t \to 0} \frac{t}{\operatorname{sen} t} = \lim_{t \to 0} \frac{1}{\frac{\operatorname{sen} t}{t}} = \frac{\lim\limits_{t \to 0} 1}{\lim\limits_{t \to 0} \frac{\operatorname{sen} t}{t}},$$

$$\Rightarrow \qquad \lim_{x \to \pi/2} \frac{\frac{1}{2}\pi - x}{\cos x} = \frac{1}{1} = 1 \qquad (\textbf{TL10}).$$

Ejercicio 7

7. $\lim_{x \to 0} \frac{\operatorname{sen}(\operatorname{sen} x)}{x}$

<u>Solución:</u>

$$\lim_{x \to 0} \frac{\operatorname{sen}(\operatorname{sen} x)}{x} = \lim_{x \to 0} \left(\frac{\operatorname{sen} x}{\operatorname{sen} x} \cdot \frac{\operatorname{sen}(\operatorname{sen} x)}{x} \right) = \lim_{x \to 0} \left(\frac{\operatorname{sen} x}{x} \cdot \frac{\operatorname{sen}(\operatorname{sen} x)}{\operatorname{sen} x} \right),$$

$$\lim_{x \to 0} \frac{\operatorname{sen}(\operatorname{sen} x)}{x} = \lim_{x \to 0} \frac{\operatorname{sen} x}{x} \cdot \lim_{x \to 0} \frac{\operatorname{sen}(\operatorname{sen} x)}{\operatorname{sen} x} = 1 \cdot 1 = 1 \qquad (\text{TL10}).$$

Ejercicio 8

8. $\lim\limits_{x \to 0} \dfrac{\cos x}{1 - \operatorname{sen} x}$

<u>Solución:</u>

$$\lim_{x \to 0} \frac{\cos x}{1 - \operatorname{sen} x} = \frac{\cos 0}{1 - \operatorname{sen} 0} = \frac{1}{1 - 0} = \frac{1}{1} = 1.$$

Ejercicio 9

9. $\lim\limits_{x \to 0} x \cot x$

<u>Solución:</u>

$$\lim_{x\to 0} x \cot x = \lim_{x\to 0} x \frac{\cos x}{\operatorname{sen} x} = \lim_{x\to 0} \frac{\cos x}{\frac{\operatorname{sen} x}{x}} = \frac{\lim\limits_{x\to 0} \cos x}{\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x}} = \frac{\cos 0}{1} = \frac{1}{1} = 1 \qquad (\text{TL10}).$$

Ejercicio 10

10. $\lim\limits_{n\to\infty} \dfrac{3^{n+1}}{1-2^{2n}}$

Solución:

$$\lim_{n\to\infty}\frac{3^{n+1}}{1-2^{2n}} = \lim_{n\to\infty}\frac{\dfrac{3^{n+1}}{2^{2n}}}{\dfrac{1-2^{2n}}{2^{2n}}} = \frac{\lim\limits_{n\to\infty}\dfrac{3^{n+1}}{2^{2n}}}{\lim\limits_{n\to\infty}\left(\dfrac{1}{2^{2n}}-1\right)} = \frac{0}{0-1} = 0 \qquad \left(2^{2n} > 3^{n+1} \text{ para } n \text{ grandes}\right)$$